
Interference-Aware Dynamic Algorithms for Energy Efficient Topology Control in Wireless Ad Hoc and Sensor Networks

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In wireless ad hoc and sensor networks, energy is a scarce resource and a considerable amount of energy is dissipated due to interference. Therefore, interference is one of the major challenges in wireless ad hoc and sensor networks. It alters or disrupts a message as it is being transmitted along a channel between source and destination. Since the messages are disrupted when the interference occurs, they have to be detected and the interfered messages have to be retransmitted. In this paper, we propose central and distributed heuristic algorithms for reducing average interference in receiver-centric interference model. In the literature, Minimum Spanning Tree (MST) algorithm is generally used through the interference coverage graph directly or indirectly in order to generate minimum average interference topology. Our algorithm, Dynamic Average Interference (DAI), however generates lower average interference as well as more sparse topology than MST. We realized that if the transmission ranges of nodes are taken into consideration at each stage of topology control algorithm, the interference of links are changed dynamically. This interference changing enables up to 22 % more energy saving than MST algorithm. Thus, DAI provides energy saving by reducing the interference as far as possible in generated topology.

Keywords: Wireless Ad-hoc and Sensor Networks, interference, topology control, average interference

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1. INTRODUCTION

In today's networks where thousands of devices may share wireless/wired links to communicate that may cause huge energy consumption and malfunction of network components, thus energy efficiency is of paramount importance in network management [1], [2], [3], [4]. Rapid developments in the last decade in wireless and hardware technologies have created multifunctional miniature wireless devices. These devices have enabled the use of wireless ad hoc and sensor networks. A wireless ad hoc network is a decentralized wireless network which does not rely on a predefined infrastructure, such as routers or access points. Instead, each node participates in routing by forwarding data for other nodes regarding dynamically changed network topology. Wireless sensor networks (WSNs) are also a kind of wireless ad hoc networks which consist of individual nodes that are able to interact with their environment by sensing or

controlling physical parameters. The aim of WSNs is to relay the sensed data to a base station (Sink) in an energy efficient way.

Nodes are battery powered in wireless ad hoc networks, thus energy consumption should be reduced to maximize the network lifetime. In a sensor node, radio component consumes considerable amount of energy. Therefore, the amount of message transmission in a network should be reduced. Interference is an unwanted signal which alters, modifies or disrupts the message as it is being transmitted along a channel between source and destination. The interference mostly occurs at the receiver during concurrent message receptions according to Signal-to-Interference Noise Ratio (SINR) threshold. In order to accomplish error-free communication, the interfered messages have to be detected and retransmitted. This process causes energy dissipation. Thus, there is a strong relationship between interference and energy dissipation.

In wireless ad hoc and sensor networks, topology control technique is generally used in order to form a sparse network, save energy and extend the network lifetime via reducing the initial topology of the network. Early studies on topology control [5], [6], [7], [8] mostly deal with the connectivity and sparseness of network while optimizing various design goals. They consider the interference reduction implicitly and if the resulting topology of topology control algorithm has low node degree, the interference is solved intuitively. However, it is proved in [9] that this intuition is inaccurate and interference of any proposed topology which considers the interference implicitly can be $\Omega(n)$ times larger than the interference of the optimum connected topology, where n is the total number of network nodes.

[10] emphasized the importance of interference reduction and started a new research thread. In [10], an explicit notion of interference is defined based on the current network traffic. However, this model requires a priori information about the traffic in a network, which is hard to obtain. Thus, a static model of interference is obviously desirable. Various static interference models and algorithms have been proposed for wireless ad hoc and sensor networks in [9], [11], [12], [13], [14], [15], [16], [17].

Although several models have also been proposed in these studies, there exist mainly two different interference models, sender-centric and receiver-centric [17] in the literature. In sender-centric model, the interference is considered to be an issue at the sender side where interference is based on the number of nodes affected by communication over a given link. On the other hand, in receiver-centric model, the interference is considered at the receiver side, where message collisions prevent proper reception. In addition, there are two different approaches for computing the interference of a network, maximum or average interference. Where the maximum interference approach computes the interference of a network considering the maximum interference of a link or a node in a network [9], [11], [12], the average interference approach computes the interference of a network considering average interference of whole links or nodes in a network [13]. The proposed algorithms in [13] for minimizing the average interference directly or indirectly use MST algorithm. Intuitively, minimizing the average interference instead of minimizing the maximum interference provides more interference reduction.

In this paper, we propose DAI algorithm and its distributed version for reducing average interference in receiver-centric interference model. DAI algorithm considers the transmission range of each node in each edge selection step. Therefore, DAI algorithm heuristically generates lower interference topology than MST. Besides, the running time of DAI algorithm is asymptotically same as MST. Our second algorithm is fully distributed in nature making it suitable for large scale applications such as sensor networks. In this

paper, we provide four main contributions and they are listed below:

- We proposed new metrics for receiver-centric interference model. To the best of our knowledge, this is the first study which identifies the average interference in receiver-centric model.
- A central heuristic algorithm is designed, theoretically analyzed and implemented in the simulation environment. Proposed algorithm outperforms MST algorithm which is generally used for minimizing average interference in literature.
- To the best of our knowledge, we propose the first distributed algorithm for the receiver-centric model.
- Although previous studies mentioned the relationship between interference and energy consumption, none of them has provided any simulations to show this relationship. To the best of our knowledge, this is the first study which provides extensive simulations about how the interference reduction affects the energy consumption. In order to accomplish this, we provide a sample WSN application.

The rest of this paper is organized as follows. In Section 2, the network representation and interference problem is described and the related work is surveyed in Section 3. The proposed interference metrics and proposed algorithms are described in Section 4 and Section 5 respectively. The results of performance tests are presented in Section 6. Lastly, conclusions are given in Section 7.

2. PROBLEM FORMULATION

2.1. Network Representation

The following assumptions are made about the network:

- Each node has distinct *node_id*.
- The nodes are stationary.
- The transmission range of a node is modeled with a circle.
- Links between nodes are symmetric. Thus if there is a link from u to v , there exists a reverse link from v to u .
- Nodes do not know their positions. They are not equipped with a position tracker like a GPS receiver.
- All nodes are equal in terms of processing capabilities, radio, battery and memory.

Based on these assumptions, wireless ad hoc and sensor networks are generally modeled as Unit Disk Graphs (UDG) [18]. In a UDG as depicted in Fig. 1, there is an edge $\{u, v\}$ if the maximum transmission radius of both u and v is at least $|uv|$, their Euclidean distance. Each node in network can adjust its transmission power to any value between zero and its maximum transmission power level. Only undirected

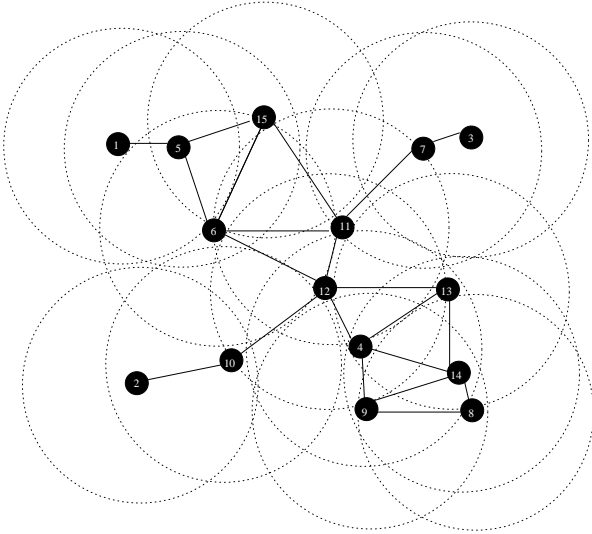


FIGURE 1. Unit disk graph model.

(symmetric) edges are considered in order to send acknowledgement over same link. The topology control algorithm aim to compute an interference optimal subgraph $G' = (V, E')$ of given $G = (V, E)$ by reducing the transmission powers of nodes.

2.2. Interference Problem

Interference problem occurs when a node's message is corrupted because of other concurrent transmissions by other nodes at the same transmission range. For a UDG modeled ad hoc network, the interference of a node is defined as the number of disks that include it. The interference causes message loss which then results high energy consumption. This situation may frequently occur in dense networks with high traffic. Thus interference is a very important problem.

The interference problem can be reduced by applying a topology control method such as a spanning tree algorithm on the communication graph. Indeed, energy saving is provided by constructing sparse topology as well as reducing interference. In order to investigate the interference problem in detail, we survey the related interference models and interference reduction methods by analyzing and comparing them.

3. RELATED WORK

In this section we firstly survey the graph theoretic and computational geometry based topology control algorithms, then we investigate the interference aware topology control models and algorithms in detail.

3.1. Graph Theoretic and Computational Geometry based Topology Control

The Delaunay triangulation [19], MST [20], the relative neighborhood graph [21], the Gabriel graph [22] are early studies on graph theoretic and computational

geometry based topology control approaches which aim to generate a sparse graph for energy efficiency.

CBTC algorithm [23] was the first attempt for construction of a topology having several desired properties such as energy spanner with bounded degree. Another aspect of topology control algorithms are gathered around the clustering. [24], [25], [26], [27], [28] are cluster-based algorithms which generally aim to construct dominating sets for clustering. They also try to minimize the energy consumption by sharing the energy dissipation among the members of clusters. The studies on topology control [5], [6], [7], [8], [19], [20], [21], [22], [24], [25], [26], [27], [28] mostly deal with the connectivity and sparseness of network while optimizing various design goals.

All graph theoretic and computational geometry based topology control algorithms mentioned so far handles the interference reduction implicitly and they claim that if the resulting graph of the topology control algorithm has low node degree, the interference is solved intuitively. However, it is proved in [9] that this intuition is inaccurate and interference of any proposed topology algorithms which emphasize the interference implicitly can be $\Omega(n)$ times larger than the interference of the optimum connected topology.

3.2. Interference Aware Topology Control

[10] emphasized the importance of interference reduction and started a new research thread. In [10], an explicit notion of interference is defined based on the current network traffic. However, this model requires a priori information about the traffic in a network, which is hard to obtain. Thus, a static model of interference is obviously desirable.

Burkhardt *et al.*[9] firstly proposed sender-centric interference model. They defined coverage of an edge as how many nodes are disturbed while nodes u and v are communicating with their transmission powers in order to measure the interference of a link. It was formally defined as;

$$Cov(e_{e \in E}) := |\{w \in V \mid w \text{ is covered by } D(u, |u, v|)\} \cup \{w \in V \mid w \text{ is covered by } D(v, |v, u|)\}|$$

Then, the maximum interference of a graph $G(V, E)$ was defined as the highest coverage edge of G . It was formally defined as;

$$MIC(G) := \max Cov(e_{e \in E})$$

LIFE, LISE and LLISE algorithms [9] are proposed for minimizing the maximum interference. LIFE minimizes the maximum interference in sender-centric model. Since LIFE can form long distance connections, a spanner algorithm, LISE, is proposed. Since our focus in this paper is minimizing the average interference, these algorithms are out of our concern.

Li *et al.*[13] proposed link-based interference model by extending the coverage definition in sender-centric model of Bukhart *et al.*. In addition, Li *et al.* defined the average interference for sender-centric model as dividing the sum of all edge coverage in G to edge count;

$$AIC(G) := \sum Cov(e_{e \in E})/|E|$$

Another interference model defined by Li *et al.*[13] was node-based. It has two different type including Node Interference via Link and Sender Centric. However, node interference via sender centric is different than the sender-centric interference defined by Burkhart *et al.*[9]. In Node Interference via Link model, the interference of a node was defined as the maximum link interference of all links incident to it. The definition is;

$$NIC(u) := \max_{x,v \in V} Cov(uv)$$

The maximum and average interference of a graph $G(V,E)$ was defined as;

$$\begin{aligned} MNIC(G) &:= \max_{u \in V} NIC_G(u) \\ ANIC(G) &:= \sum_{u \in V} NIC_G(u)/n \end{aligned}$$

In Node Interference via Sender Centric, the interference of a node $u \in V$ was defined as number of nodes inside transmission range of node u . After the topology control algorithm finishes its execution, nodes adjust their transmission powers to the minimum needed to reach its farthest neighbor in network. Let r_u denote the transmission range of node u . The definition of the interference of a node is;

$$IS(u) := |\{v \mid r_{uv} \leq r_u\}|$$

By using the definition of the $IS(u)$, the maximum and average interference of a graph $G(V,E)$ can be defined as;

$$\begin{aligned} MNIS(G) &:= \max_{u \in V} IS_G(u) \\ ANIS(G) &:= \sum_{u \in V} IS_G(u)/n \end{aligned}$$

The interference tree algorithms proposed by Li *et al.*[13] generally based on MST where interference of a link is used as an edge weight for minimizing average and maximum interference. In our paper, we propose an algorithm which outperforms the MST approach by reducing the average interference of the graph.

Johansson and Carr-Motyckova [16] proposed path based metrics for interference reduction. They defined the average path interference of a graph as the sum of interference for all interference-optimal paths between node pairs, divided by the number of all node pairs in the graph. The interference-optimal path between nodes u and v is the path $IoptP_{uv} = \{e_1, e_2, \dots, e_k\}$ between u and v that has the lowest interference, according to the following definition: the interference of a path is defined as the sum of the coverage of all

edges in the path, according to the edge coverage. The average interference of a path is formally defined as;

$$TotIoptPI(G) := \sum_{u,v \in V} \sum_{e \in IoptP_{uv}} Cov(e)$$

Then, the average interference for all shortest paths was defined as $TotSPI(G)$ divided by the number of node pairs connected in G . The average interference for all shortest paths was formally defined as;

$$TotSPI(G) := \sum_{u,v \in V} \sum_{e \in SP_{uv}} Cov(e)$$

Receiver-centric interference model was firstly defined by Fussen *et al.*[11]. This definition of interference is based on the natural question of how many nodes are affected by communication over a certain link. Therefore, the interference value of a single node u was then defined as the number of transmission powers of other nodes that include node u . It was formally defined as;

$$RI(u) := |\{u \mid u \in V \setminus \{v\}, v \in D(u, r_u)\}|$$

Fussen *et al.* defined the interference of the graph as the maximum interference of a node. In our paper, we extend this definition as the average interference. According to Fussen *et al.*, when $D(u, r_u)$ stands for the transmission circle with node u in its center and radius r_u then the interference of a graph $G(V, E)$ was then formally defined as;

$$MRI(G) := \max_{v \in V} I(v)$$

After defining receiver-centric model, Fussen *et al.* proposed Nearest Component Connector (NCC) algorithm which constructs a tree rooted at the sink node in order to minimize the maximum interference. This algorithm is out of our concern since its focus is minimization of the maximum interference.

In [12], a robust interference model for wireless ad hoc networks was proposed. It is proved in [12] that the sender-centric interference is a suspicious model. Receiver-centric is more robust model than sender-centric in terms of maximum interference of a network. Therefore, in our paper, we studied receiver-centric model instead sender-centric.

An approximation algorithm is proposed by Rickenbach *et al.* [12] in order to minimize the maximum interference in receiver-centric model on one-dimensional highway model. [15] generalizes a result given in [12] for the special case of highway model (i.e., one-dimensional problem) to the two-dimensional case. [17] presents all proposed models and algorithms in [9], [11] and [12].

4. PROPOSED INTERFERENCE METRICS

We propose two new interference metrics in order to minimize the average interference in receiver-centric

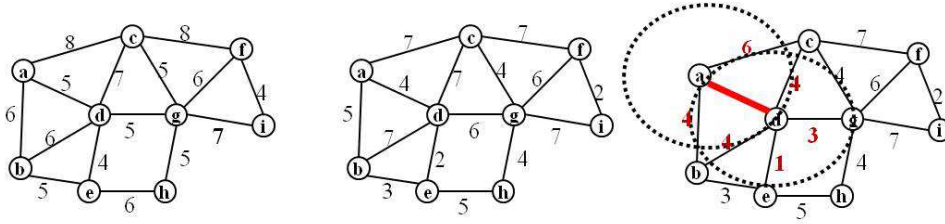


FIGURE 2. a) Distance graph b) Coverage graph c) Dynamic coverage changing.

model. Intuitively, we think that by minimizing the average interference instead of minimizing the maximum interference we may construct a more energy-efficient topology.

To the our best of knowledge, although Fussen *et. al.* [11] defined maximum interference in receiver-centric model, there is no definition for the average interference in receiver-centric interference in literature. We propose to compute the average interference in receiver-centric model by dividing the sum of node interference to node count as in Definition 1.

DEFINITION 1. Average Interference in Receiver-Centric Model:

$$ARI(G) := \sum_{u \in V} RI(u)/n$$

We use the node based via sender centric interface definition for assigning the coverage values of each node. In order to construct a interference-aware tree, we propose a new coverage definition. We defined the coverage of an edge given in Definition 2 as the sum of two nodes's interference incident to it.

DEFINITION 2. Coverage of an Edge(e):

$$Cov_{rc}(e_{e \in E}) := |\{u, v \in V | IS(u) + IS(v)\}|$$

5. DYNAMIC AVERAGE INTERFERENCE ALGORITHM (DAI)

5.1. Motivation

The proposed models and algorithms so far generally try to minimize the average or maximum interference considering the initial coverage of a link or interference of a node in a static manner. Each $e \in E$, $cov(e)$ or each $u \in V$, $RI(u)$ and $IS(u)$ is computed at the initial state, then, the algorithms perform MST construction through these values. However, this tendency may be inaccurate because proposed algorithms do not take into consideration the transmission range of each node in each edge selection step of the topology control algorithm. On the other side, interference of links may change dynamically at each edge selection step.

Fig. 2.a depicts a wireless ad hoc network as a $G(V, E)$ with edge weights denote Euclidean distance between neighboring nodes. All nodes use their maximum power in order to communicate. Fig. 2.b

depicts interference coverage graph of G which is denoted by $G_c(V, E_c)$. The coverage of edges in E_c are calculated with Cov_{rc} considering neighboring nodes use transmission power as low as they can communicate each other. For instance, the interference of node a is 1, if node d is the farthest neighbor of it. Similarly, the interference of node d is 3. According to Cov_{rc} , the coverage of edge $e=(a,d) \in E_c$ is then calculated as 4.

In order to show the above mentioned case, we give an example in Fig. 2. Assume that Prim's MST algorithm is being performed on G_c and node a is the initiator of the algorithm. In the first step, $e=(a,d) \in E_c$ is activated and added to subgraph G_c' . At this point, node a and d have to adjust their transmission ranges to 5 units in order to communicate each other in G_c' . Then, considering the current state of G_c' , the coverage of links incident to node a and d are dynamically decreased as depicted in Fig. 2.c. For instance, at this state, the coverage of $e=(c,d) \in E_c$ decreases from 7 to 4 because the transmission range of node d is 5. When we recalculate the coverage of $e=(c,d)$, node d has to increase its transmission range in order to communicate with node c , thereby, it additionally disturbs node b and e . On the other hand, node c increases its transmission range 0 to 7 in order to communicate with node d , it disturbs node d and g . Therefore, the coverage of edge $e=(c,d)$ is 4 at that state. It is concluded from this example that if the transmission power is taken into consideration, the coverage of an edge dynamically changes during the topology algorithm performance.

5.2. Central Algorithm Design and Analysis

We propose DAI algorithm, which generates low average interference for receiver-centric model. DAI algorithm simply consists two main parts. In first part, after the minimum cost node $u \in V$ is extracted, transmission range of parent of node u is adjusted and coverage of links incident to parent is updated if transmission range of parent node is changed. Then, in the second part, transmission range of node u is adjusted and coverage of links incident to node u is updated if transmission range of u is changed.

We modify the Prim's MST algorithm in [29] in order to both select the minimum edge and update the necessary edges. DAI algorithm gets interference coverage graph as input, and gives transmission range of

each node as output. In the algorithm, $d(u, v)$ returns the weight of $e \in E$. After the key , p and r_v arrays are initialized, min-priority queue Q is built according to the coverage values in key array (If same value exists, the communication cost of node will take place higher priority). r_v array stores the minimum transmission range value of node v in order to communicate with farthest neighbor in the spanning tree. p array stores the parent node of each node. key array also stores the coverage cost of each node.

Minimum cost node $u \in V$ is extracted in each iteration of the algorithm. If the parent node of u (p_u) cannot communicate with u after the spanning tree formation, transmission range of p_u is adjusted to distance $|u, p_u|$. If the transmission range of p_u is adjusted, new coverage values of links incident to p_u are recalculated and updated in min-priority queue. Similarly, node u also makes these operations.

Algorithm 1 DAI

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1: input:  $G, rt$ 
2: output:  $r_v, v \in V$ 
3: init  $key = \infty, p = NIL, r_v = NIL$  arrays
4:  $key[rt] = 0$ 
5: insert  $V$  into  $Q$  according to  $key$  values
6: while  $Q \neq \emptyset$  do
7:    $u =$  extract minimum from  $Q$ 
8:   if  $u$  is out of  $p[u]$ 's range and  $u \neq rt$  then
9:     adjust  $r_{p[u]}$  to  $d(p[u], u)$ 
10:    for all  $k \in$  neighbor of  $p[u]$  and  $k \in Q$  do
11:      Update the  $cov_{rc}(e)$  where  $e_{p[u],k} \in E$ 
12:      if  $cov_{rc} < key[k]$  then
13:         $p[v] \leftarrow u$ 
14:         $key[v] \leftarrow cov_{rc}$ 
15:      end if
16:    end for
17:  end if
18:  if  $p[u]$  is out of  $u$ 's range and  $u \neq rt$  then
19:    adjust  $r_u$  to  $d(u, p[u])$ 
20:  end if
21:  for all  $v \in$  neighbor of  $u$  and  $v \in Q$  do
22:    Update the  $cov_{rc}$  where  $e_{u,v} \in E$ 
23:    if  $cov(e) < key[v]$  then
24:       $p[v] \leftarrow u$ 
25:       $key[v] \leftarrow cov_{rc}$ 
26:    end if
27:  end for
28: end while

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Let us assume an ad hoc network with N nodes and the average interference is I . Also assume the size of a message is T and a node transmits a message with probability p within a time period of T . The average number of packets that can be sent to a node in time T is $\lambda = Ip$. The arrival of independent messages with a rate of λ can be modeled by the Poisson distribution. The probability that k packet transmissions occur within unit time T is given below:

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Assume the message sending times are not slotted then a packet x will collide if another packet is transmitting in $2T$ time. In order to successfully transmit packet x , no other packets should be transmitted within the period $2T$ as following:

$$P(0) = \frac{\lambda^0 e^{-\lambda}}{0!}$$

$$P(0) = e^{-2Ip}$$

The probability that a packet is successfully delivered after exactly k trials is given below:

$$S(k) = (1 - e^{-2Ip})^{k-1} e^{-2Ip}$$

By using the above equation, average number of trials to send a packet is driven as follows:

$$\sum_{k=1}^{\infty} (1 - e^{-2Ip})^{k-1} e^{-2Ip} = e^{2Ip}$$

The average energy consumption caused by the transmission of a packet is given below:

$$(trial\ count)((sending\ energy) + (edge\ count * receiving\ energy))$$

Assume that M is the transmission range, P_s and P_r is the reference sending and receiving power consumptions respectively, D is the edge count (both directed and undirected edge counts), the average energy consumption can be written as follows:

$$e^{2Ip}(P_s T/M^2 + DP_r T/M^2)$$

From the above equation, we may decrease the energy consumption by applying topology control in three ways: energy conservation from decreasing packet collisions caused by the interference, energy conservation from transmission range reduction, energy conservation from decreasing edge count which cause overhearing. In Theorem 5.1, we prove the average interference of DAI is at least as low as MST. In Section 6, we showed that the transmission range and the edge count performances of DAI are better than those of MST.

THEOREM 5.1. *The average interference of topology produced by DAI (ARI_{DAI}) is at least as low as those produced by MST (ARI_{MST}), thus $ARI_{DAI} \leq ARI_{MST}$.*

Proof. We prove the theorem by contradiction. Our basis assumption is average interference of topology produced by MST is lower than those produced by

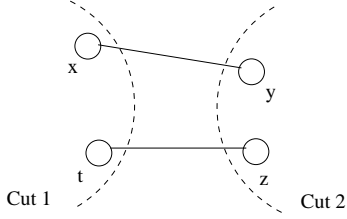


FIGURE 3. Example network with cuts.

DAI at least in one case. Assume the example in Fig. 3 where there are two cuts in the network and both algorithms choose the same edges in these cuts. Let us also assume that MST chooses e_{xy} , DAI chooses e_{tz} in order to connect these cuts. From our basis assumption, Equation 1 may be true.

$$Cov_{MST}(e_{xy}) < Cov_{MST}(e_{tz}) \quad (1)$$

Assume that $IS(a)_r$ shows the interference caused by node a when its transmission range is r , then we may extend $Cov_{MST}(e_{xy})$ as following:

$$Cov_{MST}(e_{xy}) = IS(x)_{e_{xy}} - IS(x)_0 + IS(y)_{e_{xy}} - IS(y)_0$$

$$Cov_{MST}(e_{xy}) = IS(x)_{e_{xy}} + IS(y)_{e_{xy}} \quad (2)$$

If DAI chooses e_{tz} instead of e_{xy} then $Cov_{DAI}(e_{tz}) \leq Cov_{DAI}(e_{xy})$ should be true. Using this inequality, the following should be true:

$$Cov_{MST}(e_{xy}) < Cov_{DAI}(e_{xy}) \quad (3)$$

If we extend the Equation 3 by replacing Equation 2, we obtain Equation 4 where x_p and y_p are the previous transmission ranges of node x and nodes y before selecting e_{xy} .

$$IS(x)_{e_{xy}} + IS(y)_{e_{xy}} < IS(x)_{x_p} + IS(y)_{y_p} - IS(x)_{x_p} - IS(y)_{y_p} \quad (4)$$

Even at the worst case for $x_p=0$ and $y_p=0$, this equality is not true. We contradict with our assumption. \square

THEOREM 5.2. *The running time complexity of DAI algorithm is $O(E\Delta + E\log V)$ where Δ is the neighbor set of a node with the maximum cardinality. When $\Delta \in O(\log V)$ then the time complexity is $O(E\log V)$, same as Prim's MST algorithm.*

Proof. The algorithm is analyzed considering the binary min-heap is used. Line 3-4 can be performed in a loop and it has a $O(V)$ time complexity. Since the binary min-heap is used, line 5 uses BUILD-MIN-HEAP procedure of binary min-heap and it performs this operation in $O(V)$ time complexity. The body of while loop is executed $|V|$ times. Line 7 extracts the minimum

vertex from by performing EXTRACT-MIN procedure of binary min-heap and it also performs this operation in $O(\log V)$ time. EXTRACT-MIN is executed $O(V)$ times and the total time is $O(V\log V)$. Line 8-9 perform in constant time and total is $O(V)$. The for loop in lines 10-16 are executed $O(E)$ times in worst case since the sum of the lengths of all adjacency list is $2|E|$. In Line 11, the coverage of an edge is recalculated and updated. This operation takes $O(\Delta)$ and total time is $O(E\Delta)$. Line 12-13 are executed in constant time and total is $O(E)$. Line 14 can be performed by DECREASE-KEY procedure of binary min-heap in order to update the key value in min-priority queue Q . DECREASE-KEY procedure performs $O(\log V)$ and total is $O(E\log V)$ in worst case. Line 18-20 is executed in constant time and total is $O(V)$. The for loop in lines 21-37 are executed $O(E)$ like the loop in lines 10-16. Line 21, 23 and 24 perform in constant time and total is $O(V)$. Similar to Line 11, Line 22 takes $O(E\Delta)$ time in total. Similar to Line 14, Line 25 takes $O(E\log V)$ time in total.

Thus, the total time complexity for DAI algorithm is $O(V\log V + E\Delta + E\log V + E\log V) = O(E\Delta + E\log V)$, when $\Delta \in O(\log V)$ then the time complexity is $O(E\log V)$, same as Prim's MST algorithm. \square

5.3. An Example Operation

We now demonstrate, with the help of an example, how the update of the coverage of an edge is accomplished in DAI algorithm over a sample topology displayed in Fig. 2.a.

- After $e=(a,d) \in E_c$ is activated and added to subgraph G_c' , node a and d have to adjust their transmission ranges to 5 units in order to communicate each other in G_c' .
- The coverage of links incident to node a and d are recalculated considering the possible transmission ranges of node a and d . At this point, two nodes update the coverage of links between their neighbors one by one.
- For instance, the coverage of edge $e=(a,b) \in E_c$ is recalculated and updated. If $e=(a,b)$ will be activated, node a has to adjust its transmission range from 5 to 6 units in order to communicate with node b in G_c' . Similarly, node b has to adjust its transmission range from 0 to 6 units. The coverage of edge $e=(a,b)$ ($cov_{rc}(e)$) is recalculated as;

$$\begin{aligned} x &= IS(a)_6 - IS(a)_5 \\ y &= IS(b)_6 - IS(b)_0 \\ cov_{rc}(e) &= x + y \end{aligned}$$

Therefore, the new coverage of $e=(a,b)$ is 4; node b is additionally covered by node a and node a , d and e are covered by node b if $e=(a,b)$ is activated. Node d is not included since it has been already covered by node a .

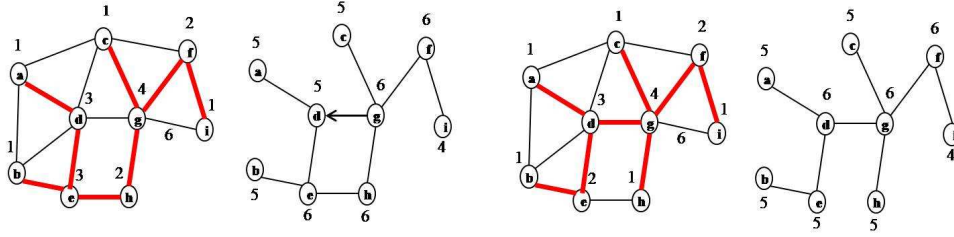


FIGURE 4. a)MST interference tree b)Generated topology by MST c)DAI interference tree d)Generated topology by DAI.

- Similarly, other edges of node a and d are updated as the update of edge $e=(a,d)$.

Fig. 4.a and Fig. 4.c show the interference trees of MST and DAI algorithms which are subgraph of sample topology displayed in Fig. 2.a respectively. The bold lines indicate the activated links. The numbers close to the vertices in Fig. 4.a and Fig. 4.c indicate the interference of a node in receiver-centric model. Fig. 4.b and Fig. 4.d depict the generated topologies of MST and DAI algorithms. The numbers close to the vertices Fig. 4.b and Fig. 4.d indicate the transmission range of a node. A line among nodes indicates unidirectional communication whereas an arrow indicates directional communication. The average interference of topology generated by MST in Fig. 4.a is $18/9=2$. On the other hand, the average interference of topology generated by DAI in Fig. 4.b is $16/9=1,7$.

5.4. Distributed Algorithm Design and Analysis

In this section, we study the distributed design and analysis of the DAI algorithm. The coverage of the links may be updated in each step of DAI, thus an asynchronous design of the algorithm is hard also it may have high message complexity due to dynamic edge updates. The message complexities of the algorithms running on ad hoc networks especially sensor networks should be as small as possible to prevent high energy consumptions. Considering these issues, we propose a semi-asynchronous distributed version of DAI algorithm. We call the designed algorithm as semi-asynchronous, since although the algorithm is divided into phases(rounds), each phase is started by a message originated from sink without using any timer mechanism. The steps of the algorithm are given in Alg. 2. We modify the Peleg's Dist.Prim algorithm [30] in order to both select the minimum edge in a phase and update the necessary edges as in DAI algorithm.

Distributed DAI Algorithm can be divided into 3 main parts. In the first part, sink node broadcasts a $START(e_{min})$ message to initiate the next phase and to inform the nodes in $MST(T)$. This information includes the new node which was selected to be included to T in the previous phase and is incident to e_{min} . In the second part of

the distributed algorithm, node x , node y and their neighbors update their coverage upon receiving $START$ message. The coverage update operation is accomplished by sending and receiving $COVERAGE$ messages. At the third part, each node finds the minimum outgoing edge(MOE) and convergencasts $MIN_EDGE(MOE)$ to the sink. In convergencast operation, a node n sends $MIN_EDGE(MOE_n)$ after receiving $MIN_EDGE(MOE_c)$ from all children in T . MOE_n is minimum value among all MOE of node n and its children.

THEOREM 5.3. *The message complexity of Distributed DAI algorithm is $\Theta(Vp)$, at the worst case is $O(V^2)$ where p is the phase count.*

Proof. In any phase, a node may send 3 messages at most: $START$, $COVERAGE$ and MIN_EDGE . Thus, the message complexity of a phase is $\Theta(V)$. For p phases the total message complexity is $\Theta(Vp)$. Worst case occurs when $p \in V$, then the message complexity is $O(V^2)$. \square

THEOREM 5.4. *The time complexity of Distributed DAI algorithm is $\Theta((D+\Delta)p)$, at the worst case is $O(V^2)$ where D is the diameter of the tree T , Δ is the maximum node degree and p is the phase count.*

Proof. The broadcast and convergencast operations take $\Theta(D)$ time. The coverage update operations need $\Theta(\Delta)$ time to complete. For p phases the total time complexity is $\Theta((D+\Delta)p)$. Worst case occurs when $p \in V$, then the time complexity is $O(V^2)$. \square

6. PERFORMANCE EVALUATIONS

In order to evaluate the average interference and energy saving, we have carried out a simulation study using ns-2 simulator. We have compared the generated topologies by MST and DAI algorithms in terms of average interference in receiver-centric model. In addition, in order to examine how energy saving is obtained, we have evaluated a WSN application on generated topologies by MST and DAI algorithms.

The initial energy of sensor nodes was set to 100 J. Random generated topologies were used in different node counts ranging from 100 to 500 nodes. We measured the performance of the algorithms for average

Algorithm 2 Distributed DAI

```

1: upon sink wakes up or receives MIN_EDGE(e) from all neighbors
2:    $e_{min} \leftarrow$  the minimum outgoing edge
3:   if  $e_{min} = \infty$  then sink terminates the algorithm by broadcasting END message
4:   else sink broadcasts START( $e_{min}$ ) message to start the next phase and to inform the nodes in T about the new node
   y and the edge  $e_{min} = (x, y)$ 
5:   end if
6: end upon
7: upon a node  $\notin \{x, y$  and their neighbors} receives START( $e_{min} = (x, y)$ )
8:   each node finds its MOE and convergecast MIN_EDGE(MOE) to the sink.
9: end upon
10: upon node x or y receives START( $e_{min} = (x, y)$ )
11:   node updates its parent in T
12:   node sets its transmission range( $R_{new}$ ) to the distance( $x, y$ )
13:   node updates the coverage for each link( $C_l$ ) according to  $R_{new}$  and  $R_{previous}$ 
14:   node sends COVERAGE( $C_L$ ) to each neighbor
15: end upon
16: upon a neighbor node(n) of x or y receives COVERAGE(C) on edge e
17:   n updates the coverage( $C_e$ ) of edge e according to C
18:   n sends COVERAGE( $C_e$ ) to the message source
19: end upon
20: upon node x or y receives COVERAGE(C) on edge e
21:    $C_E \leftarrow C$ 
22: end upon
23: upon node x, node y and their neighbors updated their coverage.
24:   each node finds its MOE and convergecasts MIN_EDGE(MOE) to the sink.
25: end upon

```

TABLE 1. Experimental study parameters.

| | |
|----------------------------|-----------------------------|
| Area | Random |
| Sink Position | Randomly placed in the area |
| Number of Sensors | 100 - 500 |
| MAC | 802.11 |
| Transmission Power | 0.660 w |
| Receiving Power | 0.395 w |
| Initial Energy | 100 J |
| Maximum Transmission Range | 250 m |
| Node degrees | 5, 8 and 11 |

node degrees varying between 5, 8 and 11. The position of sink was also set randomly. IEEE 802.11 radio and MAC standards readily available in *ns2* simulator were chosen for lower layer protocols. The initial transmission range of nodes were set to 250 m which is also the maximum transmission range of a node. The maximum transmission power and maximum receiving power were set to 0.660 w and 0.395 w respectively. The experimental study parameters are summarized in Table 1. After the MST and DAI algorithms finish their operations, the transmission and receiving power of each node was recalculated according to two-ray ground reflection model. The energy consumption parameters are given in Table 2. Using these parameters, the equations for energy consumptions during sending and receiving are given in Equation 5 and Equation 6.

$$E_{as} = \begin{cases} E_{cs}/(T_c/T_a)^2 & \text{if } T_a \leq T_c \\ E_{ms}/(T_m/T_a)^4 & \text{else} \end{cases} \quad (5)$$

TABLE 2. Energy consumption parameters.

| | |
|----------|--|
| T_c | Crossover transmission range=86.142471 m |
| T_a | Actual transmission range adjusted by topology control algorithm |
| T_m | Maximum transmission range=250.010 m |
| E_{cs} | Consumed energy while sending a message when $T_a = T_c$ |
| E_{cr} | Consumed energy while receiving a message when $T_a = T_c$ |
| E_{ms} | Consumed energy while sending a message when $T_a = T_m$ |
| E_{mr} | Consumed energy while receiving a message when $T_a = T_m$ |
| E_{as} | Actual consumed energy while sending a message |
| E_{ar} | Actual consumed energy while receiving a message |

$$E_{ar} = \begin{cases} E_{cr}/(T_c/T_a)^2 & \text{if } T_a \leq T_c \\ E_{mr}/(T_m/T_a)^4 & \text{else} \end{cases} \quad (6)$$

Fig. 5 displays the comparison of average interference of generated topologies by MST and DAI algorithms in receiver-centric model for different node counts ranging from 100 to 500 nodes. Fig. 5 shows us that DAI algorithm generates a topology which has lower average interference in receiver-centric model. DAI algorithm reduces interference compared to MST algorithm in the receiver-centric model as a percentage nearly %5 at each node count.

Fig. 6 and Fig. 7 display the undirected edge count and directed edge count difference between MST and MAI algorithms for different node counts ranging from

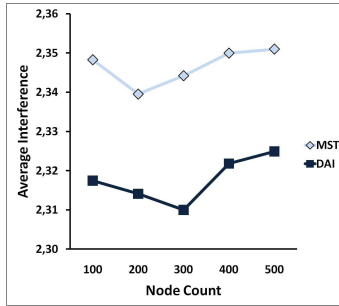


FIGURE 5. Average interference in receiver-centric interference model.

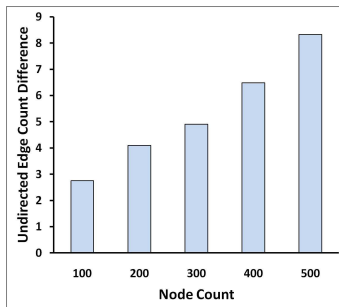


FIGURE 6. Undirected edge count differences of generated topologies by MST and DAI.

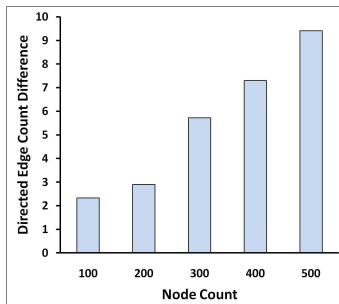


FIGURE 7. Directed edge counts differences of generated topologies by MST and DAI.

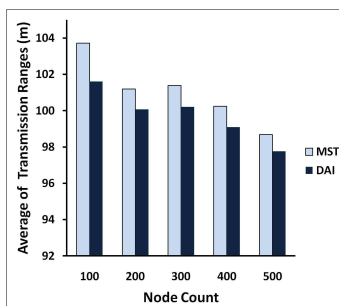


FIGURE 8. Transmission ranges of MST and DAI.

100 to 500 nodes. Both figures show us that DAI algorithm generates more sparse graph than MST algorithm. We measure the average transmission ranges as shown in Fig. 8. Although our main target is to reduce interference, the average transmission range

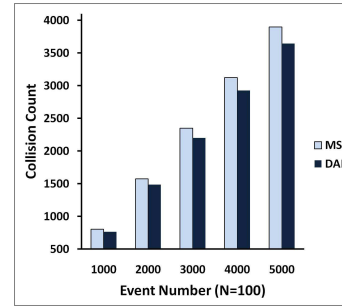


FIGURE 9. Collision counts of MST and DAI algorithms at 100 node count against varying event numbers.

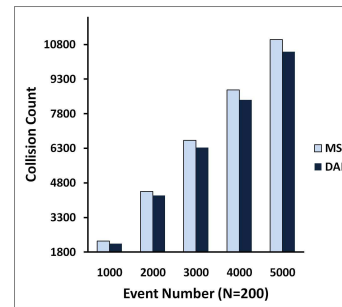


FIGURE 10. Collision counts of MST and DAI algorithms at 200 node count against varying event numbers.

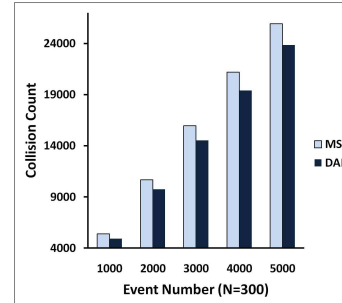


FIGURE 11. Collision counts of MST and DAI algorithms at 300 node count against varying event numbers.

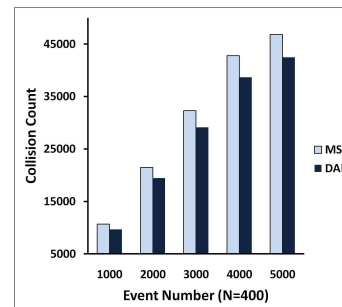


FIGURE 12. Collision counts of MST and DAI algorithms at 400 node count against varying event numbers.

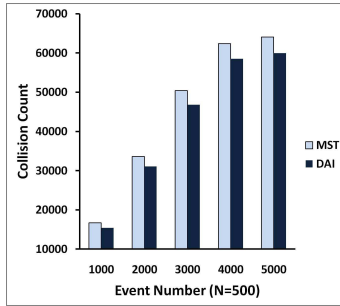


FIGURE 13. Collision counts of MST and DAI algorithms at 500 node count against varying event numbers.

produced by DAI is 0.5-1 % smaller than MST in all cases.

Fig. 9, Fig. 10, Fig. 11, Fig. 12 and Fig. 13 display the comparison of collision counts of MST and DAI algorithms against various node counts ranging from 100 to 500 nodes with fixed event number equals to 3000. At all node counts, the collision counts measured in DAI algorithm are smaller than those of MST. The collision count difference between DAI and MST reaches up to 10 % at 300 and 400 nodes.

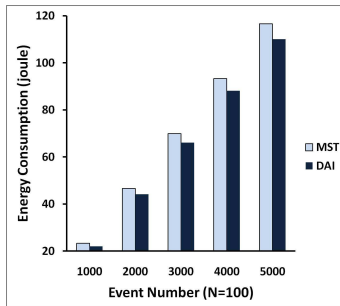


FIGURE 14. Energy consumptions of MST and DAI algorithms at 100 node count against varying event numbers.

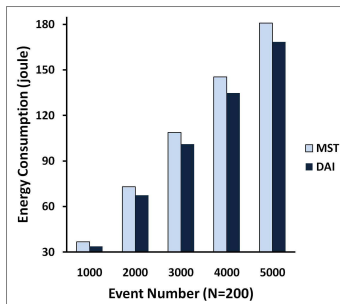


FIGURE 15. Energy consumptions of MST and DAI algorithms at 200 node count against varying event numbers.

In order to compare the energy efficiency of generated topologies by MST and DAI algorithms, we have evaluated a WSN application. In WSN application, sink

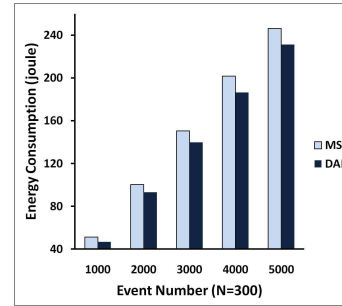


FIGURE 16. Energy consumptions of MST and DAI algorithms at 300 node count against varying event numbers.

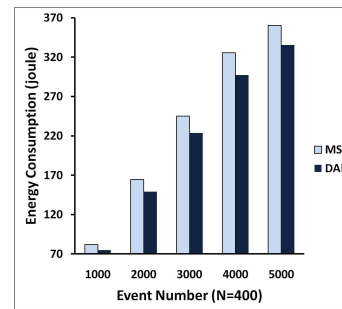


FIGURE 17. Energy consumptions of MST and DAI algorithms at 400 node count against varying event numbers.

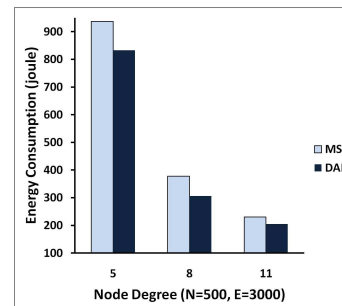


FIGURE 18. Energy consumptions of MST and DAI algorithms at 500 node count against varying event numbers.

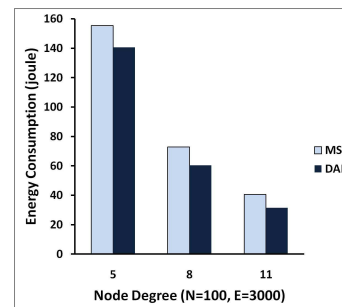


FIGURE 19. Energy consumptions of MST and DAI algorithms at 100 node count against varying degrees.

performs the MST and DAI algorithms and, assigns the

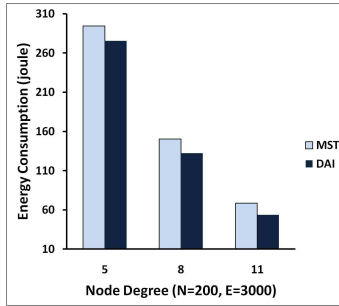


FIGURE 20. Energy consumptions of MST and DAI algorithms at 200 node count against varying degrees.

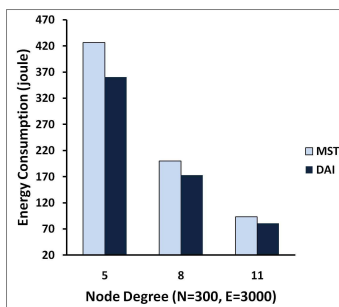


FIGURE 21. Energy consumptions of MST and DAI algorithms at 300 node count against varying degrees.

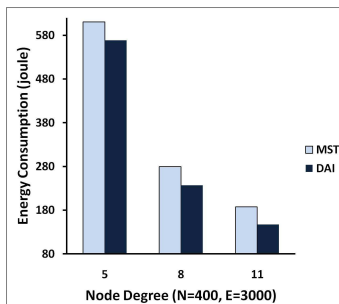


FIGURE 22. Energy consumptions of MST and DAI algorithms at 400 node count against varying degrees.

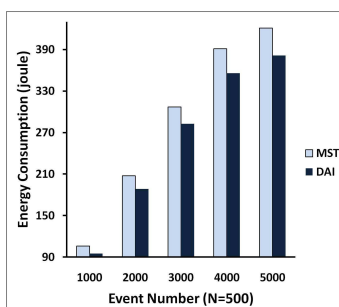


FIGURE 23. Energy consumptions of MST and DAI algorithms at 500 node count against varying degrees.

transmission and receiving power of each node. Then, each node sends message to sink through the generated spanning tree by MST and DAI algorithms one by one. We have measured the energy consumption of network

at different event numbers ranging from 2000 to 10000 events. An event is a message which contains sensed data by a sensor.

Fig. 14, Fig. 15, Fig. 16, Fig. 17 and Fig. 18 display the comparison of energy efficiency of generated topologies by MST and DAI algorithms for 100, 200, 300, 400 and 500 node counts respectively. At all node counts, DAI algorithm achieves better performance than MST algorithm in terms of energy consumption. In addition when the node count is increased, the energy savings of DAI increases. Thus, it is obvious that DAI algorithm generates more energy efficient topology than MST algorithm.

Fig. 19, Fig. 20, Fig. 21, Fig. 22 and Fig. 23 display the comparison of energy efficiency of generated topologies by MST and DAI algorithms at 5, 8 and 11 node degrees for 100, 200, 300, 400 and 500 node counts respectively. These figures show that DAI algorithm generates energy efficient topology than MST algorithm at different node degrees. When the degree equals to 11, the energy saving of DAI reaches up to 22 %.

7. CONCLUSION

Although average interference has been studied for the sender-centric model in literature, average interference in receiver-centric model has not been studied so far. In this paper, we propose DAI algorithm and its distributed version for reducing average interference in receiver-centric interference model. DAI algorithm considers the transmission range of each node in each edge selection step. Therefore, DAI algorithm heuristically generates lower interference topology than MST. Besides, the running time of DAI algorithm is asymptotically same as MST. Our second algorithm is fully distributed in nature making it suitable for large scale applications such as sensor networks. To the best of our knowledge, our proposed algorithms are the first attempts for minimizing the average interference in receiver-centric model.

In order to evaluate the average interference and energy saving, we have carried out a simulation study using ns-2 simulator. Simulation results for average interference show that DAI algorithm reduced interference compared to MST algorithm in the receiver-centric as a percentage 5%. Besides, DAI algorithm generates more sparse network than MST algorithm.

We also simulated a sample WSN application to measure the energy consumptions of topologies generated by DAI and MST algorithms. Sample WSN application results show that DAI algorithm generates more energy efficient topology than MST algorithm. We found that the energy saving of DAI reaches up to 22 % compared to the MST. From these measurements, we show the relationship between average interference

and the energy consumption, we show how the energy is saved when interference is reduced.

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